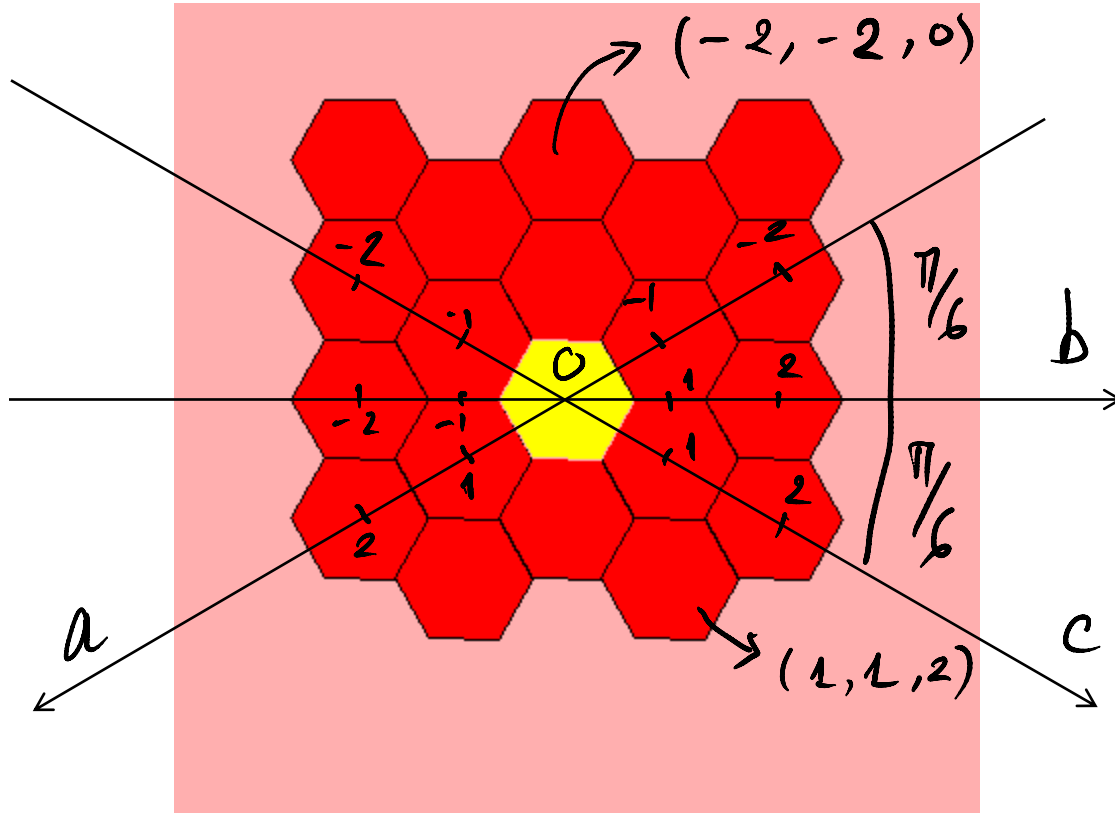The signification of this project is to create new coordinates system, called abc-coordinates system, instead of conventional xy-coordinates system. Hexagon data structure, pentahex data structure as well as game board data structure is created based on abc-coordinates system. abc-coordinates system is defined as below


- b-axis is normal x-axis in xy-coordinates system.
- a-axis creates an angle of $\frac{5\pi}{6}$ radian to b-axis
- c-axis creates an angle of π/6 radian to b-axis

Using this new abc-coordinates, each hexagon in a hexagon grid can be located by its center which has integer abc-coordinate. Above figure shows examples of this representation. Therefore, both hexagon grid and pentahex can be represented as set of hexagon centers where each hexagon center is represented by set of abc-coordinates.

Rotating hexagon around origin as well as flipping a hexagon vertically and horizontally in normal xy-coordinates system can be performed easier, faster and more efficient in abc-coordinates system. Suppose a hexagon at coordinates (a,b,c) in abc-coordinates system needs to be rotated counter-clockwise π/3 radians. New coordinates of it will be (-b,c,a). Suppose the same hexagon needs to be flipped horizontally, then its new coordinates will be (c,-b,a). Clockwise rotating and vertically flipping can be implied by the same way. Rotating and flipping a pentahex is performed by rotating and flipping individual hexagon in that pentahex.

Having (a,b,c) coordinates of hexagon centers and hexagon radius R, it is easy to calculate hexagon vertices xy-coordinates for drawing. Here, hexagon radius is defined as distance from hexagon center to its edges. Note that y-axis direction is reversed corresponding to xy-coordinates on computer screen.
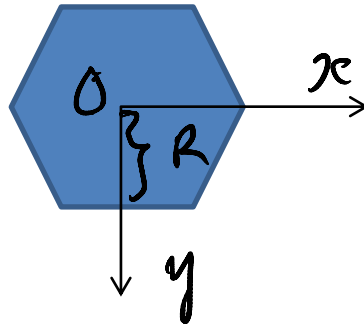
- Center of hexagon:

$$(x_c, y_c) = \left(b \times R\sqrt{3}, (a + c) \times R\right)$$

- Hexagon vertices coordinates:

$$(x_i, y_i) = \left(x_C + \frac{2R}{\sqrt{3}}\sin\left(\frac{\Pi}{6} + \frac{i\Pi}{3}\right), y_C + \frac{2R}{\sqrt{3}}\cos\left(\frac{\Pi}{6} + \frac{i\Pi}{3}\right)\right), i = 0:6$$

Xy-coordinates system is used only for drawing shapes on user screen but does not involve in other tasks such as rotating, flipping, placing, removing shapes, scoring and recognizing winning condition. These tasks are completed using abc-coordinates. In conclusion, data structures used in the game programming are created based on abc-coordinates. A pentahex or a game board consists of many hexagons where each hexagon is represented by its radius and its center location on abc-coordinates. These center location coordinates are integers. Rotating and flipping a pentahex is achieved by simple operations.